TEMPERATURE DISTRIBUTION ON THE SURFACE OF A SEMIINFINITE BODY IN THE PRESENCE OF A THIN ANNULAR HEAT SOURCE

V. P. Kozlov

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We solve the two-dimensional time-dependent heat-conduction problem for a system of three cylindrical regions heated by a time-dependent heat flux through an annular region on the surface.

In the last few years a great deal of attention, both in the domestic and the foreign literature [1-9], has been paid to the study of the heat exchange in a semiinfinite (in the thermal sense) body with local heat sources of various geometrical shapes on the surface. The solution of heat-conduction problems of this type has been treated in great detail in the early fundamental treatise [10]. Increased interest in such time-dependent heat-conduction problems for semiinfinite bodies has taken place because the solutions of these problems can be used in theoretical and practical applications.

The first area of application is to mathematical physics and the theory of special functions such as degenerate hypergeometric functions, Whittaker and Kummer functions, parabolic cylinder functions, etc. [11-14] which are generated by second-order ordinary differential equations into which the heat equation separates in the case of a half-space with discontinuous boundary conditions of the second kind.

Secondly, the time-dependent local sources and sinks of thermal energy acting on the surface of a semiinfinite body are typical of many thermal processes in electronic devices, since each diode or transistor mounted into the body is a time-dependent local source of heat on the surface of the body. Therefore, an accurate calculation of the optimal thermal regime of an electronic device leads to a thermal problem of the type discussed above [15].

Thirdly, the time-dependent development of spatial temperature fields due to a supply of heat to a portion of the surface of the body through a local region (in the shape of a circle, ring, strip, etc.) serves as the mathematical foundation of working methods and devices for the nondestructive control of thermophysical characteristics of materials (i.e, without destroying the material) [1-9].

In the present paper we consider the problem for the two-dimensional time-dependent temperature field $\Theta_i$ $(r, z, \tau)$ $(i = 1, 2, 3)$ in cylindrical coordinates on the surface $z = 0$ of a semiinfinite body when there is an arbitrary heat flux density $q(\tau)$ acting in an annular region $R_2 \geq r \geq R_1$ on the surface of the body (see Fig. 1). In the regions $r < R_1$ and $\infty > r > R_2$ we assume that the surface $z = 0$ is thermally insulated. The initial temperature distribution inside the body is assumed to be constant, $T_0$ = const. The origin of the coordinate system $r = z = 0$ is chosen at the center of the annular region bounded by $r = R_1$ and $r = R_2$ $(R_2 > R_1)$ on the surface $z = 0$. The thermophysical characteristics of the semiinfinite body are assumed to be constant and independent of temperature.

The mathematical formulation of the thermophysical problem reduces to a system of three heat equations of the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial\Theta_i(r,\ z,\ \tau)}{\partial r}\right]+\frac{\partial^2\Theta_i(r,\ z,\ \tau)}{\partial z^2}=\frac{1}{a}\frac{\partial\Theta_i(r,\ z,\ \tau)}{\partial\tau}\quad(i=1,\ 2,\ 3). \tag{1}$$

The ranges of the variables r, z, $\tau$ are:

$$\Theta_1(r,\ z,\ \tau)=T_1(r,\ z,\ \tau)-T_0\ (0\leqslant r<R_1,\ z\geqslant 0,\ \tau>0);$$
$$\Theta_2(r,\ z,\ \tau)=T_2(r,\ z,\ \tau)-T_0\ (R_1<r<R_2,\ z\geqslant 0,\ \tau>0);$$
$$\Theta_3(r,\ z,\ \tau)=T_3(r,\ z,\ \tau)-T_0\ (R_2<r\leqslant\infty,\ z\geqslant 0,\ \tau>0).$$

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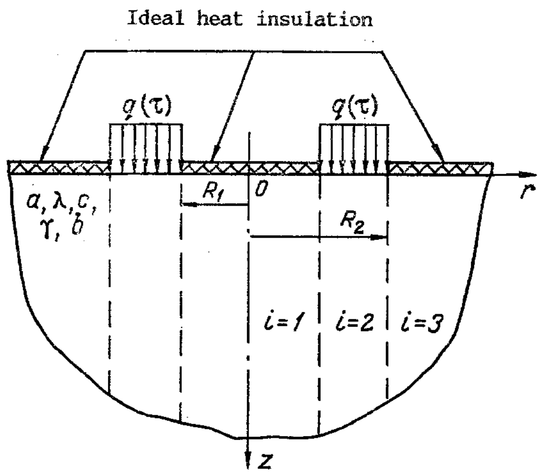


Fig. 1. Model of a thermally semi-infinite body heated through an annular region on the surface by a heat flux density q(τ).

The initial condition for system (1) is

$$\Theta_i(r,\ z,\ 0) = 0, \tag{2}$$

and the boundary conditions are

$$-\lambda \frac{\partial \Theta_2(r,\ 0,\ \tau)}{\partial z} = q(\tau) \quad (R_1 < r < R_2); \tag{3}$$

$$\frac{\partial \Theta_1(r,\ 0,\ \tau)}{\partial z} = \frac{\partial \Theta_3(r,\ 0,\ \tau)}{\partial z} = 0 \begin{pmatrix} 0 \leqslant r < R_1, \\ r > R_2 \end{pmatrix}; \tag{4}$$

$$\frac{\partial \Theta_1(0,\ z,\ \tau)}{\partial r} = 0; \tag{5}$$

$$\frac{\partial \Theta_i(r,\ \infty,\ \tau)}{\partial z} = 0; \tag{6}$$

$$\frac{\partial \Theta_3(\infty,\ z,\ \tau)}{\partial r} = 0; \tag{7}$$

$$\Theta_1(R_1,\ z,\ \tau) = \Theta_2(R_1,\ z,\ \tau); \tag{8}$$

$$\frac{\partial \Theta_1(R_1,\ z,\ \tau)}{\partial r} = \frac{\partial \Theta_2(R_1,\ z,\ \tau)}{\partial r}; \tag{9}$$

$$\Theta_2(R_2,\ z,\ \tau) = \Theta_3(R_2,\ z,\ \tau); \tag{10}$$

$$\frac{\partial \Theta_2(R_2,\ z,\ \tau)}{\partial r} = \frac{\partial \Theta_3(R_2,\ z,\ \tau)}{\partial r}. \tag{11}$$

The solution of (1) with the boundary conditions (2)-(11) can be obtained with the help of Fourier and Laplace transform methods and can be written in the following form for z = 0:

$$\Theta_1(r,\ 0,\ \tau) = \sqrt{\frac{2}{\pi}}\, \frac{1}{b} \sum_{n=0}^{\infty} \sum_{m=0}^{n} A_{n,m} \left(\frac{r}{R_2}\right)^{2n} \left(\frac{R_2}{\sqrt{a}}\right)^{2n-m-\frac{1}{2}} \int_0^{\tau} q(\tau-\xi)\, \xi^{-n+\frac{m}{2}-\frac{1}{4}} \left[\left(\frac{R_1}{R_2}\right)^{-m-\frac{1}{2}} \times \right.$$
$$\times \exp(-R_1^2/8a\xi)\, W_{\frac{2n-m}{2}+\frac{1}{4},\ \frac{m}{2}-\frac{1}{4}} \left(\frac{R_1^2}{4a\xi}\right) - \exp(-R_2^2/8a\xi) \times \tag{12}$$
$$\left. \times\, W_{\frac{2n-m}{2}+\frac{1}{4},\ \frac{m}{2}-\frac{1}{4}} \left(\frac{R_2^2}{4a\xi}\right)\right] d\xi \quad (0 \leqslant r < R_1);$$

$$\Theta_2(r,\ 0,\ \tau)=\frac{1}{b\sqrt{\pi}}\int_0^\tau\frac{q(\tau-\xi)}{\sqrt{\xi}}d\xi-\sqrt{\frac{2}{\pi}}\frac{1}{b}\sum_{n=0}^{\infty}\sum_{m=0}^{n}A_{n,m}\times$$

$$\times\left(\frac{r}{R_2}\right)^{2n}\left(\frac{R_2}{\sqrt{a}}\right)^{2n-m-\frac{1}{2}}\int_0^\tau q(\tau-\xi)\,\xi^{-n+\frac{m}{2}-\frac{1}{4}}\exp(-R_2^2/8a\xi)\times$$

$$\times W_{\frac{2n-m}{2}+\frac{1}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{R_2^2}{4a\xi}\right)d\xi-\sqrt{\frac{2}{\pi}}\frac{R_1}{\lambda}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{A_{n,m}}{2(n+1)}\times \tag{13}$$

$$\times\left(\frac{R_1}{r}\right)\left(\frac{R_1}{\sqrt{a}}\right)^{2n}\left(\frac{r}{\sqrt{a}}\right)^{-m-\frac{1}{2}}\int_0^\tau q(\tau-\xi)\,\xi^{-n+\frac{m}{2}-\frac{3}{4}}\exp(-r^2/8a\xi)\times$$

$$\times W_{\frac{2n-m}{2}+\frac{3}{4},\ \frac{m}{2}+\frac{1}{4}}\left(\frac{r^2}{4a\xi}\right)d\xi\quad(R_1<r<R_2);$$

$$\Theta_3(r,\ 0,\ \tau)=\sqrt{\frac{2}{\pi}}\frac{R_2}{\lambda}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{A_{n,m}}{2(n+1)}\left(\frac{R_2}{r}\right)\left(\frac{R_2}{\sqrt{a}}\right)^{2n}\times$$

$$\times\left(\frac{r}{\sqrt{a}}\right)^{-m-\frac{1}{2}}\left[1-\left(\frac{R_1}{R_2}\right)^{2n+2}\right]\int_0^\tau q(\tau-\xi)\,\xi^{-n+\frac{m}{2}-\frac{3}{4}}\exp(-r^2/8a\xi)\times \tag{14}$$

$$\times W_{\frac{2n-m}{2}+\frac{3}{4},\ \frac{m}{2}+\frac{1}{4}}\left(\frac{r^2}{4a\xi}\right)d\xi\quad(R_2<r<\infty).$$

Here $b = \lambda/\sqrt{a}$ ($W\cdot sec^{1/2}/m^2\cdot deg$) is the thermal activity of the semiinfinite body,

$$A_{n,m}=\frac{C_n^m\left(\frac{1}{2}\right)_m 2^m}{4^n(n!)^2}=\frac{(2m-1)!!}{4^n n!m!(n-m)!}, \tag{15}$$

where $1\cdot3\cdot5\cdot7\ \ldots\ (2m-1)=\pi^{-1/2}2^m-1/2)/2^m$ is the Pokhammer symbol, $(2m-1)!!=1\cdot3\cdot5\cdot7\ \ldots\ (2m-1)=\pi^{-1/2}2^m\,\Gamma(m+1/2)$ is the odd factorial, and the $C_n^m$ are the binomial coefficients.

The Whittaker functions $W_{k,\mu}(X) = W_{k,-\mu}(X)$ are members of the class of degenerate hypergeometric functions [11-14] and (using various notations) can be expressed in terms of the Kummer functions $U(a,\ c,\ X) \equiv \Psi(a,\ c;\ X)$ or $M(a,\ c,\ X) = \Phi(a,\ c;\ X)$, i.e., $W_{k,\mu}(X) = \exp(-X/2)X^{1/2+\mu}U(1/2+\mu-k,\ 1+2\mu,\ X)$ or $W_{k,\mu}(X) = \exp(-X/2)\times X^{1/2+\mu}\{\Gamma(-2\mu)M(1/2+\mu-k,\ 1+2\mu,\ X)/\Gamma(1/2-\mu-k) + X^{-2\mu}\times\Gamma(2\mu)M(1/2-\mu-k,\ 1-2\mu,\ X)/\Gamma(1/2+\mu-k)\}$; the generalized hypergeometric series of Gauss ${}_2F_0(\alpha,\ \beta;\ -1/X)$, i.e., $W_{k,\mu}(X) = \exp(-X/2)X^k{}_2F_0(1/2-k+\mu,\ 1/2-k-\mu,\ -1/X)$; the E-function of MacRobert $E(\alpha,\ \beta::X)$, i.e., $W_{k,\mu}(X) = \exp(-X/2)\,X^kE(1/2+\mu-k,\ 1/2-\mu-k::X)/\Gamma(1/2+\mu-k)\Gamma(1/2-\mu-k)$ and other functions (see Table 1).

If the inner radius $R_1$ of the annular heater goes to zero ($R_1 \to 0$) we obtain from (13) and (14) (the solution (12) for $\Theta_1(r,\ 0,\ \tau)$ vanishes, since in the limit $R_1 \to 0$ the region $0 \le r \le R_1$ reduces to a single point) the time-dependent temperature distribution on the surface of the body as a function of the radius r and time $\tau > 0$ when there is a thin circular heat source of radius $R_2$ on the surface:

$$\lim_{R_1\to0}\Theta_2(r,\ 0,\ \tau)=\Theta_2^*(r,\ 0,\ \tau)=\frac{1}{b\sqrt{\pi}}\int_0^\tau\frac{q(\tau-\xi)}{\sqrt{\xi}}d\xi-\sqrt{\frac{2}{\pi}}\frac{1}{b}\times$$

$$\times\sum_{n=0}^{\infty}\sum_{m=0}^{n}A_{n,m}\left(\frac{r}{R_2}\right)^{2n}\left(\frac{R_2}{\sqrt{a}}\right)^{2n-m-\frac{1}{2}}\int_0^\tau q(\tau-\xi)\,\xi^{-n+\frac{m}{2}-\frac{1}{4}}\exp(-R_2^2/8a\xi)\,W_{\frac{2n-m}{2}+\frac{1}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{R_2^2}{4a\xi}\right)d\xi$$

$$(0\leqslant r<R_2); \tag{16}$$

$$\lim_{R_1\to0}\Theta_3(r,\ 0,\ \tau)=\Theta_3^*(r,\ 0,\ \tau)=\sqrt{\frac{2}{\pi}}\frac{R_2}{\lambda}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{A_{n,m}}{2(n+1)}\times$$

$$\times\left(\frac{r}{R_2}\right)^{-m-\frac{3}{2}}\left(\frac{R_2}{\sqrt{a}}\right)^{2n-m-\frac{1}{2}}\int_0^\tau q(\tau-\xi)\,\xi^{-n+\frac{m}{2}-\frac{3}{4}}\exp(-r^2/8a\xi)\ W_{\frac{2n-m}{2}+\frac{3}{4},\ \frac{m}{2}+\frac{1}{4}}(r^2/4a\xi)\,d\xi$$

$$(R_2<r\leqslant\infty). \tag{17}$$

TABLE 1. Special Cases and Relations for the Whittaker Functions $W_{k,\mu}(X)$

| No. | $W_{k,\mu}(X)$: $k$ | $W_{k,\mu}(X)$: $\pm\mu$ | Relation | Function |
|---|---|---|---|---|
| 1 | $k$ | $\mu$ | $e^{-\frac{X}{2}} X^{\frac{1}{2}+\mu} \times U\left(\frac{1}{2}+\mu-k,\ 1+2\mu,\ X\right)$ | Kummer |
| 2 | $k$ | $\mu$ | $\frac{\Gamma(-2\mu)}{\Gamma\left(\frac{1}{2}-\mu-k\right)} M_{k,\mu}(X) + \frac{\Gamma(2\mu)}{\Gamma\left(\frac{1}{2}+\mu-k\right)} M_{k,-\mu}(X)$ | Whittaker |
| 3 | $k$ | $\mu$ | $e^{-\frac{X}{2}} X^k {}_2F_0\left(\frac{1}{2}-k+\mu,\ \frac{1}{2}-k-\mu,\ -\frac{1}{X}\right)$ | Degenerate hypergeometric function |
| 4 | $k$ | $\mu$ | $\frac{X^k}{\Gamma\left(\frac{1}{2}-k-\mu\right)\Gamma\left(\frac{1}{2}-k+\mu\right)} \times E\left(\frac{1}{2}-k-\mu,\ \frac{1}{2}-k+\mu::X\right)$ | MacRobert E function |
| 5 | 0 | $\mu$ | $\sqrt{\frac{X}{\pi}} K_\mu\left(\frac{X}{2}\right)$ | Modified Bessel function |
| 6 | $\frac{\nu}{2}+\frac{1}{4}$ | $\frac{1}{4}$ | $2^{-\frac{\nu}{2}} X^{\frac{1}{4}} D_\nu(\sqrt{2X})$ | Parabolic cylinder or Weber–Hermite function |
| 7 | 0 | $n+\frac{1}{2}$ | $\sqrt{\frac{X}{\pi}} K_{n+\frac{1}{2}}\left(\frac{X}{2}\right)$ | Spherical Bessel |
| 8 | 0 | $\frac{1}{3}$ | $2\sqrt{\pi}\left(\frac{3}{4}X\right)^{\frac{1}{6}} \mathrm{Ai}\left[\left(\frac{3}{4}X\right)^{\frac{2}{3}}\right]$ | Airy |
| 9 | $\frac{\alpha+1}{2}+n$ | $\frac{\alpha}{2}$ | $e^{-\frac{X}{2}} X^{\frac{(\alpha+1)}{2}} (-1)^n n! L_n^{(\alpha)}(X)$ | Laguerre |
| 10 | $\frac{a-1}{2}$ | $\frac{a}{2}$ | $e^{\frac{X}{2}} X^{\frac{(1-a)}{2}} \Gamma(a,\ X)$ | Incomplete gamma function |
| 11 | $-\frac{1}{2}$ | 0 | $e^{\frac{X}{2}} X^{\frac{1}{2}} E_1(X)$ | Exponential integral |
| 12 | $\frac{\nu}{2}$ | $\frac{1}{2}$ | $\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right) k_\nu\left(\frac{X}{2}\right),\quad X>0$ | Bateman |
| 13 | $\frac{n}{2}+\frac{1}{4}$ | $\frac{1}{4}$ | $e^{-\frac{X}{2}} X^{\frac{1}{4}} 2^{-n} H_n(\sqrt{X})$ | Hermite |
| 14 | $-\frac{1}{4}$ | $\frac{1}{4}$ | $\sqrt{\pi} X^{\frac{1}{4}} e^{\frac{X}{2}} \mathrm{erfc}(\sqrt{X})$ | Probability integral |
| 15 | $-\frac{3}{4}$ | $\frac{1}{4}$ | $2\sqrt{\pi} X^{\frac{1}{4}} e^{\frac{X}{2}} \mathrm{ierfc}(\sqrt{X})$ | Multiple probability integral |
| 16 | $\frac{1}{4}$ | $\frac{1}{4}$ | $X^{\frac{1}{4}} e^{-\frac{X}{2}}$ | |
| 17 | $\frac{3}{4}$ | $\frac{1}{4}$ | $X^{\frac{3}{4}} e^{-\frac{X}{2}}$ | |

If the outer radius of the annular heater goes to infinity ($R_2 \to \infty$) then we obtain from (12) and (13) (the solution (14) for $\Theta_3(r, 0, \tau)$ vanishes since when $R_2 \to \infty$ the region $R_2 < r \leq \infty$ is infinitely far from the surface) the time-dependent temperature distribution on the surface of a semiinfinite body as a function of the radius r and time $\tau > 0$ when there is a time-dependent heat source of strength $q(\tau)$ acting in the region $R_1 < r \leq \infty$ on the surface of the body, and in the region $0 \leq r < R_1$ ($z = 0$) the heat flux vanishes along the normal to the boundary of the body, i.e., in this region the body is thermally insulated:

$$\lim_{R_2\to\infty}\Theta_1(r,\ 0,\ \tau) = \Theta_1^{**}(r,\ 0,\ \tau) = \sqrt{\frac{2}{\pi}}\,\frac{1}{b}\sum_{n=0}^{\infty}\sum_{m=0}^{n} A_{n,m}\left(\frac{r}{R_1}\right)^{2n}\times$$
$$\times\left(\frac{R_1}{\sqrt{a}}\right)^{2n-m-\frac{1}{2}}\int_0^{\tau} q(\tau-\xi)\,\xi^{-n+\frac{m}{2}-\frac{1}{4}}\exp(-R_1^2/8a\xi)\times \tag{18}$$
$$\times W_{\frac{2n-m}{2}+\frac{1}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{R_1^2}{4a\xi}\right)d\xi \quad (0 \leqslant r < R_1);$$

$$\lim_{R_2\to\infty}\Theta_2(r,\ 0,\ \tau) = \Theta_2^{**}(r,\ 0,\ \tau) = \frac{1}{b\sqrt{\pi}}\int_0^{\tau}\frac{q(\tau-\xi)}{\sqrt{\xi}}\,d\xi - \sqrt{\frac{2}{\pi}}\times$$
$$\times\frac{R_1}{\lambda}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{A_{n,m}}{2(n+1)}\left(\frac{r}{R_1}\right)^{-m-\frac{3}{2}}\left(\frac{R_1}{\sqrt{a}}\right)^{2n-m-\frac{1}{2}}\int_0^{\tau} q(\tau-\xi)\,\xi^{-n+\frac{m}{2}-\frac{3}{4}}\exp(-r^2/8a\xi)\times \tag{19}$$
$$\times W_{\frac{2n-m}{2}+\frac{3}{4},\ \frac{m}{2}+\frac{1}{4}}\left(\frac{r^2}{4a\xi}\right) \quad (R_1 < r \leqslant \infty).$$

The solutions (12)-(19) have the correct limiting behavior. For example, if we put $R_1 = 0$ in (19) we obtain the one-dimensional result of [16]. From (16) evaluated at $r = 0$, we obtain the particular solution of [5], relating the excess temperature $\Theta_2^*(0, 0, \tau)$ at the center ($r = z = 0$) to the heat flux density acting through the surface $z = 0$ and inside the circular region $r = R_2$, since the Whittaker function for the first term of the double sum ($n = m = 0$) will be equal to

$$W_{\frac{1}{4},\ \frac{1}{4}}\left(\frac{R_2^2}{4a\xi}\right) = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{a}}{R_2}\right)^{-\frac{1}{2}}\xi^{-\frac{1}{4}}\exp(-R_2^2/8a\xi).$$

We now consider the application of the time-dependent temperature fields (12)-(19) for specific forms of $q(\tau - \xi)$. We note that these temperature fields are complicated functions of several variables but the prime issue is the explicit dependence of the excess temperatures $\Theta_i(r, 0, \tau)$ (on the surface of the body) on all of the thermophysical characteristics of the semiinfinite body. This can be used to implement numerous "pure" nonstationary methods of nondestructive control of the thermophysical characteristics of materials, i.e., the entire set of thermophysical characteristics can be determined without inserting temperature sensors into the body.

Which of the solutions given here is best to use in studying the thermophysical characteristics of materials depends on the inventiveness of the researcher and his ability to realize in practice the boundary conditions postulated theoretically and methods to supply controllable heat fluxes and to measure the corresponding temperatures.

## NOTATION

$\Theta_i(r, 0, \tau)$, excess temperature on the surface of the semiinfinite body in the three regions of r discussed in the text; $R_2$, $R_1$, r, the outer and inner radii of the annular heater and the radius vector magnitude; $q(\tau)$, arbitrary time-dependent heat flux density in a given local region of heating of a source; a, $\lambda$, b, thermal diffusivity, thermal conductivity, and thermal activity of the semiinfinite body; z, $\tau$, cylindrical coordinate and time; $A_{n,m}$, constant thermal amplitudes (see text); $W_{k,\mu}(X)$, Whittaker function; $U(a, c, X) = \Psi(a, c; X)$, $M(a, c, X) = \Phi(a, c; X)$, Kummer functions; ${}_2F_0(\alpha, \beta; -1/X)$, generalized hypergeometric function; $E(\alpha, \beta::X)$, E-function of MacRobert; $\Gamma(a)$, the gamma function;

$\Theta_i^*(r, 0, \tau), \Theta_i^{**}(r, 0, \tau)$, $T_0$, excess temperatures on the surface of the semiinfinite body in the limiting cases for $R_1$ and $R_2$ (see text) and the initial temperature, respectively.

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# CALCULATION OF AN OPTICAL SYSTEM WITH A HOLLOW MIRROR LIGHTGUIDE AND DIAPHRAGMS FOR PHOTOELECTRIC DEVICES


V. B. Rantsevich



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A calculation method and nomograms are presented for optical systems with a hollow mirror cylindrical lightguide, input and output diaphragms, and a radiation receiver.


Hollow mirror lightguides [1, 2] are now being used in photoelectric equipment, especially pyrometers, together with lenses, mirrors, lightguides made of opticaly transparent materials, and other elements. The hollow mirror guides are nonselective, simple in construction, convenient in use, have high mechanical strength, and are low in cost. However, no methods are available for calculation of an optical system with hollow lightguides interacting with other elements — diaphragms, radiation receivers, lenses, etc.


Applied Physics Institute, Academy of Sciences of the Belorussian SSR, Minsk.


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